Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 9 1 4 6 9 4 9 6 4 0 *

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **October/November 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 **(a)** Sketch the graph of $y = |4x - 2|$. [1]

(b) Solve the inequality $1 + 3x < |4x - 2|$. [4]

2 The parametric equations of a curve are

$$x = (\ln t)^2, \qquad y = e^{2-t^2},$$

for $t > 0$.

Find the gradient of the curve at the point where $t = e$, simplifying your answer. [4]

3 The polynomial $2x^3 + ax^2 - 11x + b$ is denoted by $p(x)$. It is given that $p(x)$ is divisible by $(2x - 1)$ and that when $p(x)$ is divided by $(x + 1)$ the remainder is 12.

Find the values of a and b. [5]

4 **(a)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 4 - 3\mathrm{i}| \leqslant 2$ and $\mathrm{Re}\, z \leqslant 3$. [4]

(b) Find the greatest value of $\arg z$ for points in this region. [2]

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5 Find the exact value of $\int_0^6 \frac{x(x+1)}{x^2+4}\,dx$. [6]

6 **(a)** By sketching a suitable pair of graphs, show that the equation

$$\cot x = 2 - \cos x$$

has one root in the interval $0 < x \leqslant \frac{1}{2}\pi$. [2]

(b) Show by calculation that this root lies between 0.6 and 0.8. [2]

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(c) Use the iterative formula $x_{n+1} = \tan^{-1}\left(\frac{1}{2 - \cos x_n}\right)$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 (a) By expressing 3θ as $2\theta + \theta$, prove the identity $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$. [3]

(b) Hence solve the equation

$$\cos 3\theta + \cos\theta\cos 2\theta = \cos^2\theta$$

for $0° \leqslant \theta \leqslant 180°$. [5]

8 It is given that $\dfrac{2 + 3a\mathrm{i}}{a + 2\mathrm{i}} = \lambda(2 - \mathrm{i})$, where a and λ are real constants.

(a) Show that $3a^2 + 4a - 4 = 0$. [4]

(b) Hence find the possible values of a and the corresponding values of λ. [3]

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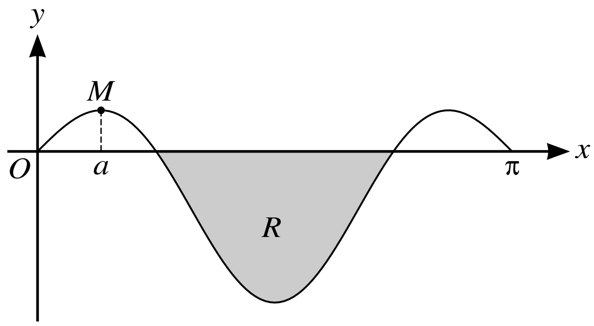

The diagram shows the curve $y = \sin x \cos 2x$, for $0 \leqslant x \leqslant \pi$, and a maximum point M, where $x = a$. The shaded region between the curve and the x-axis is denoted by R.

(a) Find the value of a correct to 2 decimal places. [5]

(b) Find the exact area of the region R, giving your answer in simplified form. [4]

10 The equations of the lines l and m are given by

$$l: \quad \mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \qquad \text{and} \qquad m: \quad \mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 4 \\ c \end{pmatrix},$$

where c is a positive constant. It is given that the angle between l and m is 60°.

(a) Find the value of c. [4]

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(b) Show that the length of the perpendicular from (6, −3, 6) to l is $\sqrt{11}$. [5]

11 The variables x and y satisfy the differential equation

$$x^2 \frac{dy}{dx} + y^2 + y = 0.$$

It is given that $x = 1$ when $y = 1$.

(a) Solve the differential equation to obtain an expression for y in terms of x. [8]

(b) State what happens to the value of y when x tends to infinity. Give your answer in an exact form. [1]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.